

# Continuous Wave Function Collapse in Quantum-Electrodynamics?

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**Abstract.** Time-continuous wavefunction collapse mechanisms *not* restricted to markovian approximation have been found only a few years ago, and have left many issues open. The results apply formally to the standard relativistic quantum-electrodynamics. I present a generalized Schrödinger equation driven by a certain complex stochastic field. The equation reproduces the *exact* dynamics of the interacting fermions in QED. The state of the fermions appears to collapse continuously, due to their interaction with the photonic degrees of freedom. Even the formal study is instructive for the foundations of quantum mechanics and of field theory as well.

## INTRODUCTION

The first study of what we can call continuous wave function collapse was Mott's analysis [1] of the particle's track in a cloud chamber. The advanced models [2]-[4] of real continuous collapse have become part of the quantum optics theoretical toolbox. Speculations on fictitious or formal continuous wave function collapse started with [5] by Bohm and Bub and have been discussed in numerous works [6]-[18] of various motivations. As for the mathematical formalisms of both real and fictitious collapses, the triumphing one turned out to be the markovian Stochastic(ally modified) Schrödinger Equation (SSE) [9],[12],[13]. The Lorentz invariant SSE is a hard nut, see e.g. [19] and references therein, as well as [20],[21]. In my opinion, a sensible Lorentz invariant SSE should relax the markovian approximation first. After my early attempts [22]-[24], Strunz published an important result [25] to be followed soon by many others [26]-[33]. I am going to recall the standard markovian and non-markovian SSE, and I shall present and discuss a SSE in the explicit Lorentz invariant context of QED.

## REAL OR FICTITIOUS CONTINUOUS COLLAPSE

Classicality emerges from Quantum via real or fictitious, often time-continuous, measurement (detection, observation, monitoring, e.t.c.) of the wavefunction  $\psi$ . By the real continuous collapse we mean, e.g., the detection of a particle track in a cloud chamber, the photon-counter detection of atomic decay, or the homodyne detection of quantum-optical oscillators. The fictitious continuous collapse means the various theories of spontaneous (universal, intrinsic, primary, e.t.c.) collapse (localization, reduction, e.t.c.). The items in the parentheses indicate the multitude of close synonyms. To date, the mathe-

matics is the same for both real and fictitious classes! We know almost everything about the mathematical and physical structures if markovian approximation applies. We know much less beyond that approximation. So, let us see what *equation* describes the wave function under time-continuous collapse?

## THE MARKOVIAN STOCHASTIC SCHRÖDINGER EQUATION

The prototype of the SSE has the following structure:

$$\begin{aligned} \frac{d\psi(t,z)}{dt} = & \begin{aligned} & -i\hat{H}\psi(t,z) && \text{hermitian Hamiltonian} \\ & -i\hat{q}z\psi(t,z) && \text{non-hermitian noisy Hamiltonian} \\ & -\frac{1}{2}\gamma\hat{q}^2\psi(t,z) && \text{non-hermitian dissipative Hamiltonian,} \end{aligned} \end{aligned} \quad (1)$$

where  $z$  is a complex Gaussian hermitian white-noise:

$$\mathbf{M}[z^*(t)z(s)] = \gamma\delta(t-s). \quad (2)$$

Throughout this work,  $\mathbf{M}[\dots]$  stands for the stochastic average. The eq.(1) is not norm-preserving. We define the physical state by  $\psi/\|\psi\|$  while its statistical weight must be multiplied by  $\|\psi\|^2$ :

$$\psi(t,z) \longrightarrow \frac{\psi(t,z)}{\|\psi(t,z)\|} \equiv |t,z\rangle, \quad (3)$$

$$\mathbf{M}[\dots] \longrightarrow \mathbf{M}[\|\psi(t,z)\|^2 \dots] \equiv \tilde{\mathbf{M}}_t[\dots]. \quad (4)$$

In our case, the state  $|t,z\rangle$  and the noise  $z(t)$  play the roles of the Quantum and the Classical, respectively. Their “mutual influence” is described by the eqs.(1-4)<sup>1</sup>.

The markovian SSE describes perfectly the time-continuous collapse of the wave-function in the given observable(s)  $\hat{q}$ . The state  $|t,z\rangle$  depends on  $\{z(s); s \leq t\}$  causally, i.e., the state at  $t$  depends on the values of the noise at times  $s \leq t$ . The individual solutions  $|t,z\rangle$  can, in principle, be realized by time-continuous monitoring of  $\hat{q}$ . Then  $z(t)$  becomes the classical record explicitly related to the monitored value of  $\hat{q}$ .

Our key-problems will be: causality, realizability, and Lorentz invariance. So far, for the markovian SSE, both causality and realizability hold but Lorentz invariance is missing. To make a progress, we need to relax the markovian approximation.

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<sup>1</sup> Equivalently, there exists a pair of closed non-linear equations for  $|t,z\rangle$  and for the recorded value of  $\hat{q}$ , cf. [12],[13], or [18].

# THE NON-MARKOVIAN STOCHASTIC SCHRÖDINGER EQUATION

The key to the non-markovian SSE is that the driving noise is a colored noise:

$$\mathbf{M}[z^*(t)z(s)] = \alpha(t-s). \quad (5)$$

The corresponding SSE [26] contains a memory-term:

$$\frac{d\psi(t,z)}{dt} = -i\hat{H}\psi(t,z) - i\hat{q}z\psi(t,z) + i\hat{q}\int_0^t \alpha(t-s)\frac{\delta\psi(t,z)}{\delta z(s)}ds. \quad (6)$$

This linear SSE is not norm-preserving. We define the physical state by  $\psi/\|\psi\|$  while its statistical weight must be multiplied by  $\|\psi\|^2$  exactly the same way as in eqs.(3,4) for the markovian SSE<sup>2</sup>.

The non-markovian SSE describes the *tendency* of time-continuous collapse of the wavefunction in the given observable(s)  $\hat{q}$ . The state  $|t,z\rangle$  depends on  $\{z(s); s \leq t\}$  causally. The individual solutions  $|t,z\rangle$  can *not* be realized by any known way of monitoring [33]. The non-markovian SSE corresponds mathematically to the influence of a real or fictitious oscillatory reservoir whose instantaneous Husimi function is sampled stochastically, cf. [28]. Disappointedly,  $z(t)$  can *not* be interpreted as a classical record. It only corresponds to fictitious paths in the parameter space of the reservoir's coherent states.

The status of our key problems for the non-markovian SSE is the following. Causality holds, but realizability and Lorentz invariance may not hold at all. Can we enforce Lorentz invariance at least?

## CASE STUDY: QUANTUM-ELECTRODYNAMICS

We choose the well-known quantum theory of electromagnetic interaction as the framework to study the possible form of a Lorentz invariant SSE — without any guarantee that it exists. At least, we shall try to export the Lorentz invariance from QED to SSE.

Let  $x = (x_0, \vec{x})$  denote the four-vector of space-time coordinates. The vector-field  $\hat{A}(x)$  stands for the quantized electromagnetic four-potential, and the Dirac spinor field  $\hat{\chi}(x)$  stands for the quantized electron-positron field. Then the fermionic current is defined by  $\hat{J}(x) = e\hat{\bar{\chi}}(x)\gamma\hat{\chi}(x)$ . Later we shall need the electromagnetic correlation  $D(x) = i\langle \text{e.m.vac} | \hat{A}(x)\hat{A}(0) | \text{e.m.vac} \rangle$  as well. The Schrödinger equation in interaction picture takes this form:

$$\frac{d\Psi(t)}{dt} = -i \int_{x_0=t} d\vec{x} \hat{J}(x) \hat{A}(x) \Psi(t). \quad (7)$$

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<sup>2</sup> There exists a non-linear non-markovian SSE for  $|t,z\rangle$  [27]. There is no equation for anything like the recorded value of  $\hat{q}$ . To date, there has been no way to define a classical record.

As usual in QED, we suppose the uncorrelated initial state  $\Psi(-\infty) = \psi(-\infty) \otimes |\text{e.m.vac}\rangle$  where  $\psi(-\infty)$  is the initial state of the electrons and positrons before the interaction is switched on. We seek the SSE for the electron-positron wavefunction  $\psi(t)$  continuously localized by the electromagnetic field which plays the role of the “environment” or the “reservoir”. It can be shown that the SSE is driven by the negative-frequency part  $A^-(x)$  of the e.m. “vacuum-field”  $A^+ + A^- = A$ , satisfying:

$$\mathbf{M}[A^-(x)A^+(y)] = \langle \text{e.m.vac} | \hat{A}(x)\hat{A}(0) | \text{e.m.vac} \rangle = -iD(x-y). \quad (8)$$

The SSE<sup>3</sup> contains a memory-term:

$$\frac{d\psi(t, A^-)}{dt} = -i \int_{x_0=t} d\vec{x} \hat{J}(x) A^-(x) \psi(t, A^-) - \int_{x_0=t} d\vec{x} \int_{y_0 \leq t} dy \hat{J}(x) D(x-y) \frac{\delta \psi(t, A^-)}{\delta A^-(y)}. \quad (9)$$

This linear non-markovian SSE is not norm-preserving. We define the physical state by  $\psi/\|\psi\|$  while its statistical weight must be multiplied by  $\|\psi\|^2$ , cf. eqs.(3,4):

$$\psi(t, A^-) \longrightarrow \frac{\psi(t, A^-)}{\|\psi(t, A^-)\|} \equiv |t, A\rangle, \quad (10)$$

$$\mathbf{M}[\dots] \longrightarrow \mathbf{M}[\|\psi(t, A^-)\|^2 \dots] \equiv \tilde{\mathbf{M}}_t[\dots]. \quad (11)$$

Similarly to the markovian SSE, the state  $|t, A\rangle$  and the random complex field  $A^-(x)$  play the role of the Quantum and the Classical, respectively. Their “mutual influence” is described by the eqs.(8-11) which are *formally* Lorentz invariant. The solutions of the “relativistic” SSE (9), when averaged over  $A^-$ , describe the exact QED fermionic reduced state:

$$\mathbf{M}[\psi(t, A^-) \psi^\dagger(t, A^+)] = \text{tr}_{\text{e.m.}}[\Psi(t) \Psi^\dagger(t)]. \quad (12)$$

The “relativistic” SSE describes the *tendency* of time-continuous collapse of the fermionic wavefunction in the current  $\hat{J}$  although the collapse happens in (certain) Fourier components instead of the local values  $\hat{J}(x)$ . The state  $|t, A\rangle$  depends on the classical field  $\{A(x); x_0 \leq t\}$  causally. The individual solutions  $|t, A\rangle$  can *not* be realized by any known way of monitoring. Therefore the classical field  $A$  can *not* be interpreted as a classical record<sup>4</sup>.

## LORENTZ INVARIANCE?

We can express the solution of the “relativistic” SSE (9), emerging from the initial state  $\psi(-\infty)$ :

$$\psi(t, A^-) = \text{T exp} \left\{ -i \int_{x_0 \leq t} d\vec{x} \hat{J}(x) A^-(x) - \int \int_{y_0 \leq x_0 \leq t} d\vec{x} dy \hat{J}(x) D(x-y) \hat{J}(y) \right\} \psi(-\infty), \quad (13)$$

<sup>3</sup> This equation follows from the results of [26]-[28], where a closed non-markovian SSE for the normalized state  $|t, A\rangle$  is also given.

<sup>4</sup> The complex random field  $A^-$  carries information on the quantized e.m. field  $\hat{A}(x)$ , the details are still to be investigated.

where  $T$  stands for time-ordering of the current operators  $\hat{J}(x)$ . Consider the expectation value of the local e.m. current at some  $t$ :

$$J(t, \vec{x}, A) = \frac{\psi^\dagger(t, A^+) \hat{J}(t, \vec{x}) \psi(t, A^-)}{\psi^\dagger(t, A^+) \psi(t, A^-)}. \quad (14)$$

This local current  $J(x, A)$  depends on  $A(y)$  for  $y_0 \leq x_0$  which is causal in the given frame while it may violate causality in other Lorentz frames. To assure Lorentz invariant causality, the local current  $J(x, A)$  must not depend on  $A$  outside the backward light-cone of  $x$ . Since this is not the case, our “relativistic” SSE can not be causal at all.

We can see that, despite our efforts, the status of the key-problems for the “relativistic” SSE is disappointing: causality, realizability, and Lorentz invariance have been all lost. However, the “relativistic” SSE is the prototype of a Lorentz-invariant-looking closed SSE and we must study it if we wish to know why and where exactly Lorentz invariance has gone<sup>5</sup>.

## OUTLOOK

“Classicality emerges from Quantum via real or fictitious, often time-continuous, measurement (detection, observation, monitoring, e.t.c.) of the wavefunction  $\psi$ .” This has been our universal motivation to investigate the corresponding mathematical models. All markovian SSE’s turn out to be mathematically equivalent with standard (though sophisticated) quantum measurements [18]. The non-markovian SSE’s are equivalent with certain quantum reservoir dynamics, i.e., with their formal stochastic decompositions (unravelings) [26]. We have inspected in the previous section that the causality and Lorentz invariance (as well as the realizability) remain problematic even when we start from a true Lorentz invariant dynamics.

Let us ask the following question. Can we construct more general models which would liberate us from the mathematical constraints of the standard quantum theory? The minimalist’s answer would be this. We should replace the concept “Emergence of Classicality from Quantum” by the concept “Coexistence of Classical and Quantum”. The classical entities<sup>6</sup> are certain classical fields  $C(x)$  and the quantum entities are certain quantum fields  $\hat{Q}(x)$ . We seek a causal and Lorentz invariant coexistence including their “mutual influence” on each other. The loophole is that, unlike in the quantum theory, the “mutual influence” is not necessarily a dynamical or a measurement-like mechanism. In our longstanding struggles with the problem of Classical vs. Quantum, the main issue to overcome has always been the painful lack of a consistent model that “couples” the coexisting classical and quantum entities. Aren’t quantum dynamics and measurement too restrictive? Are there any other consistent mechanisms?

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<sup>5</sup> Lorentz invariance would already be lost in a single von Neumann-Lüders collapse. Nevertheless, the status of Lorentz invariance in continuous collapse is a theoretical challenge.

<sup>6</sup> According to certain alternative concepts [12],[21],[34],[35] the whole physics might be represented by classical entities so that we may not care if the wave function violates Lorentz invariance and causality.

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